The signature of edge states in the energy level distribution of a 2DEG in a strong magnetic field

1994 J. Phys.: Condens. Matter 6 4737

(http://iopscience.iop.org/0953-8984/6/25/012)

J. Phys.: Condens. Matter 6 (1994) 4737–4744. Printed in the UK

The signature of edge states in the energy level distribution of a 2DEG in a strong magnetic field

D Eliyahu and R Berkovits

The Jack and Pearl Resnick Institute for Advanced Technology, Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

Received 3 March 1994, in final form 10 May 1994

Abstract. The level spacing distributions of a confined 2DEG (2D electron gas) in a strong magnetic field are studied. As long as the Landau bands do not overlap the distribution statistics exhibit a sharp transition in the vicinity of the edge states. The level spacing distribution between the pair of edge states follow the Poisson statistics while the level spacing in their vicinity is non-universal.

The statistical properties of the energy spectrum of disordered electronic systems have been the subject of many investigations [1–14]. These properties are usually described in terms of the eigenvalue properties of random matrices, first used by Wigner and Dyson to describe the spectrum properties of complex nuclei [15–17]. For a system which is metallic the level spacing follows a particular form of Wigner statistics (GOE, GUE or GSE) depending on the symmetry of the system. The level spacing statistics are given by

$$\begin{aligned} P_{\mathrm{GOE}}(s) &= \frac{\pi s}{2}\exp(-\pi s^2/4) \\ P_{\mathrm{GUE}}(s) &= \frac{32 s^2}{\pi^2}\exp(-4s^2/\pi) \\ P_{\mathrm{GSE}}(s) &= \frac{2^{18}s^4}{(9\pi)^3}\exp(-64s^2/9\pi) \end{aligned} \tag{1}$$

where s is the energy separation between two consecutive levels in units of the mean level spacing Δ. For a metallic system which has a time reversal symmetry GOE level spacing statistics is observed, while for a system where time reversal symmetry is broken (for example in the presence of a magnetic field) GUE level spacing statistics emerge. GSE level spacing distribution follows for cases in which spin–orbit scattering is present. Once the system is localized Poisson statistics,

$$P_{\mathrm{P}}(s) = \exp(-s) \tag{2}$$

is observed.

Recently, many studies on the crossover between the different statistical ensembles have been performed. A crossover between GOE and Poisson distributions as a function of disorder was observed by several authors [7–9]. In the crossover region the distribution is described as a superposition of GOE and Poisson sequences [7, 8], or using a phenomenological distribution which in the limiting cases goes to GOE or Poisson distributions [9]. In a

0953-8984/94/254737+08$19.50

recent theoretical study Kravtsov *et al* [10] suggest that at the mobility edge the distribution should take a new form which depends on dimensionality. The crossover between the GOE and the GUE distributions as a function of an AB (Aharonov–Bohm) flux was studied by Dupuis and Montambaux [11]. The crossover region is fitted to a distribution obtained by Pandry and Metha [18] where the crossover distribution is studied using the combination of symmetric and antisymmetric 2×2 matrices.

For a 2DEG in the presence of a strong magnetic field the situation is even more complicated. One expects that close to the centre of the Landau band the distribution will correspond to GUE statistics since the states for a finite sample are expected to be extended and the time reversal symmetry is broken [12–14]. In the tails the states are localized, therefore, the level spacing statistics follow Poisson. Numerical investigations of the lowest Landau level performed by Ono *et al* [13, 14] show that the pure GUE form is never really obtained since for short trajectories, which determine the tail of the distribution, time reversal is not completely broken and the tail of the distribution corresponds to GOE. This effect is even more pronounced moving out of the centre of the Landau band, where the distribution is fitted to a crossover from GUE to GOE form.

In this paper we shall mainly consider the behaviour of the level spacing distribution near the tail of the Landau band. In contrast to the previous work [12–14] we shall consider a system which has a confining hard wall potential namely the Laughlin geometry. In the Laughlin geometry the 2D system is represented by a cylinder of circumference L_x and height L_y subject to a transverse constant magnetic field H as is shown in figure 1. The energy spectrum of such a system has several states in between the Landau bands, commonly referred to as edge states. Those states have several interesting properties. The electron density is concentrated along the edge of the sample, but the wavefunction is extended in the direction parallel to the edge, i.e., it may carry current in the $\hat{x}$ direction once a flux threads the system. Classically these states correspond to trajectories bouncing off the edge. Those states interact very weakly with phonons [19, 20] and have a very long inelastic scattering time. We shall see that the special properties of edge states will lead to a non-trivial behaviour of the level spacing distributions in their vicinity.

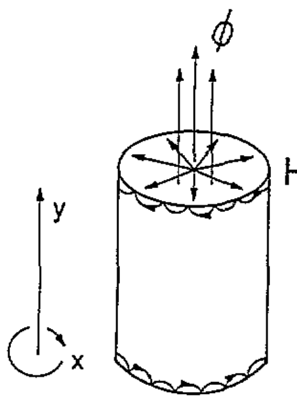

Figure 1. The sample geometry. A two-dimensional cylinder with circumference L_x and height L_y. The metal is threaded by a static magnetic flux ϕ in the y direction and transverse uniformly magnetic field H normal to the sample area. The classical trajectories of edge states bouncing off the cylinder's edge are illustrated.

For non-interacting electrons, the system may be described by the Hamiltonian

$$\hat{H} = \frac{1}{2m}\left[\hat{p} - \left(\frac{e}{c}Hy\right)e_x\right]^2 + \hat{V}(r) \tag{3}$$

where e_x denotes a unit vector in the x direction. The random potential $V(r)$ is assumed to be a Gaussian-distributed white noise, defined by

$$\langle V(r)\rangle = 0 \qquad \langle V(r)V(r')\rangle = \gamma\delta(r - r') \tag{4}$$

where $\langle\ldots\rangle$ denotes averaging over realizations of a disorder potential, $\gamma = \hbar v_F/2\pi\ell N(\mu)$, $N(\mu)$ is the averaged density of states at the Fermi energy μ, v_F is the Fermi velocity and ℓ is the elastic mean free path. For $y = 0$, L_y hard-wall boundary conditions are assumed while in the longitudinal direction periodic boundary conditions $(x, x + L_x)$ are taken into account.

For the numerical calculations we shall use a tight-binding Hamiltonian in the Landau gauge which represents the same system [21–23]

$$H = \sum_{k,j} \epsilon_{k,j} a_{k,j}^{\dagger} a_{k,j} - V \sum_{k,j} (\exp(i\theta(k)) a_{k,j+1}^{\dagger} a_{k,j} + \mathrm{HC}) - V \sum_{k,j} (a_{k+1,j}^{\dagger} a_{k,j} + \mathrm{HC}) \tag{5}$$

where $\epsilon_{k,j}$ is the energy of a site located in the jth row (x axis) and kth column (y axis), which is chosen randomly between $-W/2$ and $W/2$, V is a constant hopping matrix element. The phase $\theta(k) = 2\pi Hks^2/\phi_0$ (s is the distance between sites and $\phi_0 = hc/e$ is the flux quantum) stems from the transverse magnetic field. This Hamiltonian is then exactly diagonalized, and the energy spectrum is then obtained.

The typical sample sizes in the numerical calculation are 10×20 sites. The level spacing distribution was usually calculated for 500 different realizations of disorder, where the disorder strengths were chosen as $W = V$ and $2V$. The level spacing distribution was calculated for different values of the magnetic field $H = 0.2$ and $0.4\phi_0/s^2$. Since one flux per lattice plaquet (where the distance between tight-binding sites is estimated for a typical density of carriers in a 2DEG $\rho \sim 4 \times 10^{11}$ cm^{-2} as $s \sim 1/\sqrt{\rho} \sim 200$ Å) corresponds to a magnetic field of about 10 T, these values of magnetic field are within the usual range used in quantum Hall effect experiments.

Typical numerical results for the averaged energy levels are presented in figure 2. The spectrum is symmetric around $E = 0$ because of the basic particle–hole symmetry of this model. The edge states are clearly seen in the gap between the bulk states as can be expected from the appearance of the Landau quantization. The mean energy level spacing Δ depends strongly on the level number, and drastically changes around the edge states.

The states close to the centre of any Landau band obey the GUE distribution except for the tail of the distribution which follows a GOE like behaviour. This is in agreement with the results of Ono *et al* [13, 14] which were obtained for the lowest Landau level and periodic boundary conditions. As the levels approach the edge of the band the distribution acquires more GOE characteristics.

In figure 3 we present a typical sequence of level spacing distributions between the first and second Landau bands. It is obvious that there are strong changes in the level spacing distributions between consecutive levels. One can see that for some of the levels the distribution follows Poisson (levels 38–39, 40–41), for others GOE (42–43), and for others the distribution is not at all clear (37–38, 39–40, 41–42). Our aim is to explain this complicated behaviour of the distributions in the tail region between the Landau bands.

As a starting point we chose to study the case of a rather strong magnetic field ($H = 0.4\phi_0/s^2$). The DOS (density of states) in the absence of disorder is shown in figure 4(a). Once disorder is introduced into the system, clear Landau bands appear in the DOS (figure 4(b)). The edge states clearly appear in the DOS of the disordered system as a small peak in the DOS in between the Landau bands.

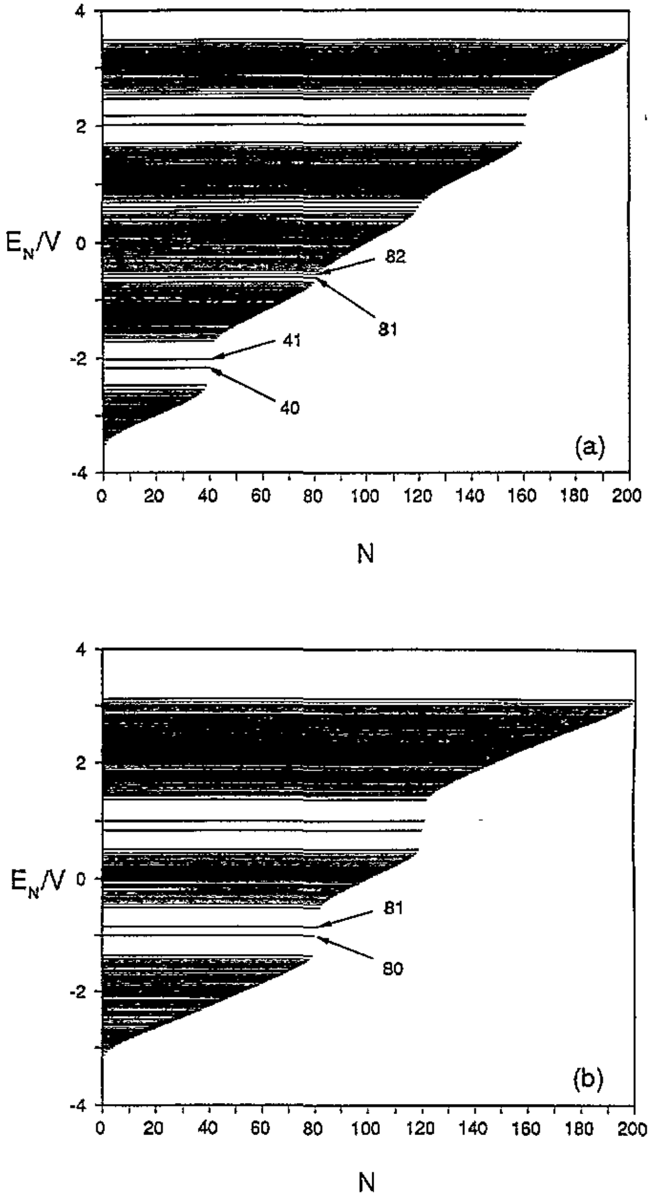

Figure 2. The numerical results of the energy spectrum E_N as a function of the level number for a 10×20, $W = 2V$ sample in the presence of a magnetic field. (a) $H = 0.2\phi_0/s^2$, (b) $H = 0.4\phi_0/s^2$.

The level spacing distribution for this case is presented in figure 5. The distribution between the original edge states (levels 80–81) follows an almost perfect Poisson distribution. This stems from the fact that in contrast to the bulk states, the pair of edge states, although extended in the $\hat{x}$ direction, are localized near the edges of the sample one at $y = 0$ and the second at $y = L_y$. Therefore, the overlap between the eigen-functions is extremely small, resulting in a Poisson distribution. This is an interesting twist of the usual situation in disorder systems where extended current carrying levels follow a GOE-

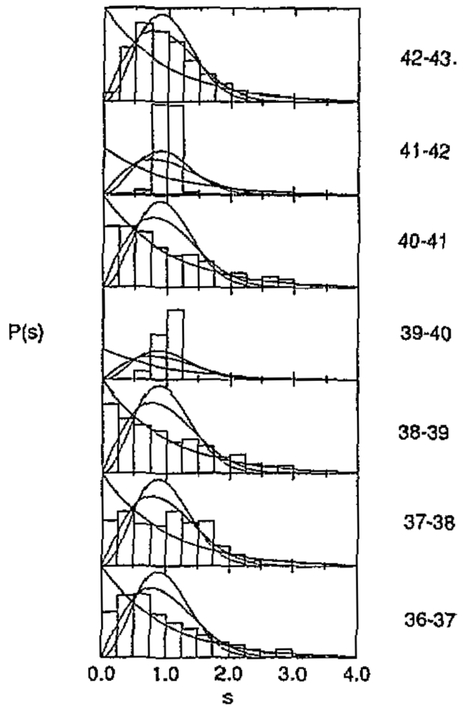

Figure 3. The level spacing distribution for different energy levels for a $W = V$, $H = 0.2\phi_0/s^2$ sample.

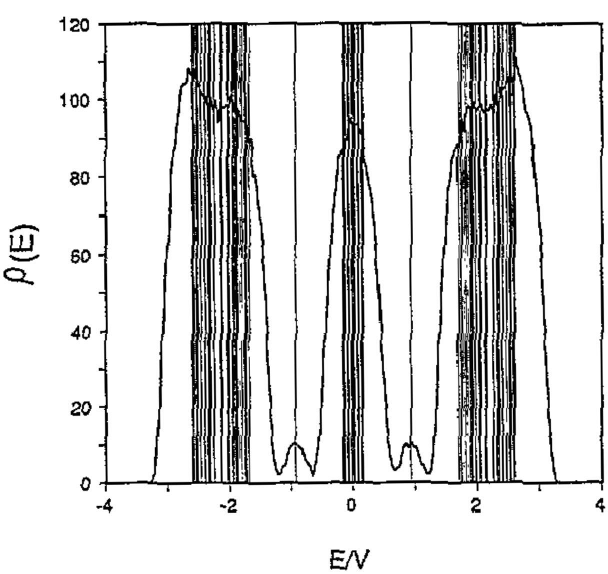

Figure 4. Numerical results for the DOS for a weakly disordered sample $W = 2V$, $H = 0.4\phi_0/s^2$. The vertical lines represent the positions of the energy levels in the ordered case ($W = 0$).

GUE distribution. This can be also understood from considering the classical trajectories corresponding to the edge states illustrated in figure 1. Those trajectories which constantly bounce off the edge are expected to exhibit a rather regular behaviour, which from the postulate of 'quantum chaos' [24, 25] should lead to a Poisson distribution.

The spacing between the edge states and their adjacent levels (levels 79–80, 81–82) is

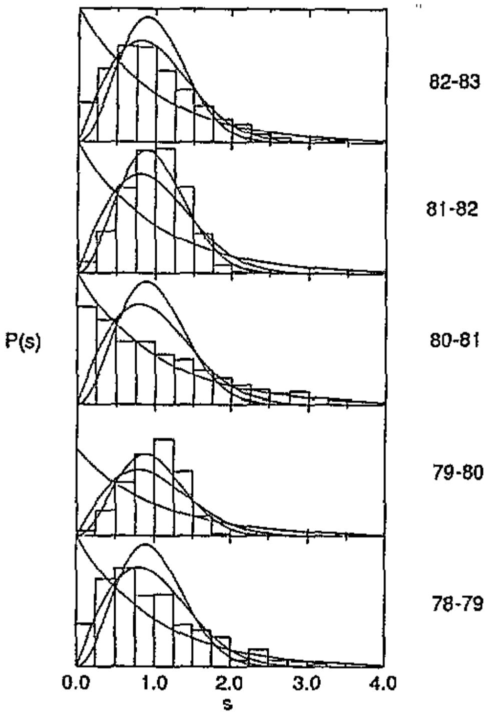

Figure 5. The level spacing distribution for different energy levels around the edge states (see figure 2(b)) for a $W = 2V$, $H = 0.4\phi_0/s^2$ sample.

non-universal. This non-universality is the remanent of the original gap between the edge states and the Landau bands which is still clearly seen in the DOS of the system (figure 4(a)). The levels which belong to the Landau bands (levels 78–79, 82–83) exhibit a GOE Poisson crossover behaviour typical of states in the tail of the band [13, 14].

This typical distribution structure also provides the explanation for the complicated structure seen in figure 3. Levels 38, 39 and 40, 41 are edge states, and therefore their spacing distribution follows Poisson statistics. The spacings between different couples of edge states, and between them and other Landau band states is non-universal which explains the unusual distributions seen for levels 37–38, 39–40 and 41–42. The last pair of levels (42–43) belongs to the Landau band and have the expected GOE distribution.

As the two Landau bands start to overlap, which may be caused by broadening due to a stronger disorder or due to the fact that one considers higher bands, the edge states are encompassed by the bands and lose their special characteristic. This may be clearly seen in figure 6(a) where the level spacing distributions for levels between the first and second Landau bands are presented and in figure 6(b) where the distributions for levels between the second and third band are shown. In the first case the two bands do not overlap and the situation is similar to that for figure 5, where levels 40, 41 are the pair of edge states. For the second case, the original pair of edge states (81, 82) are almost an integral part of the Landau bands (as can be seen in figure 2(a)) and have the regular GOE–Poisson crossover behaviour.

In conclusion, for confined electron systems one should be careful in calculating the level spacing distributions near the tails of the Landau band. The presence of edge states in that region causes strong fluctuations in the distributions between consecutive levels. Level spacings between pairs of edge states follow a Poisson distribution, while the spacing between an edge state and Landau band states or higher edge states are non-universal.

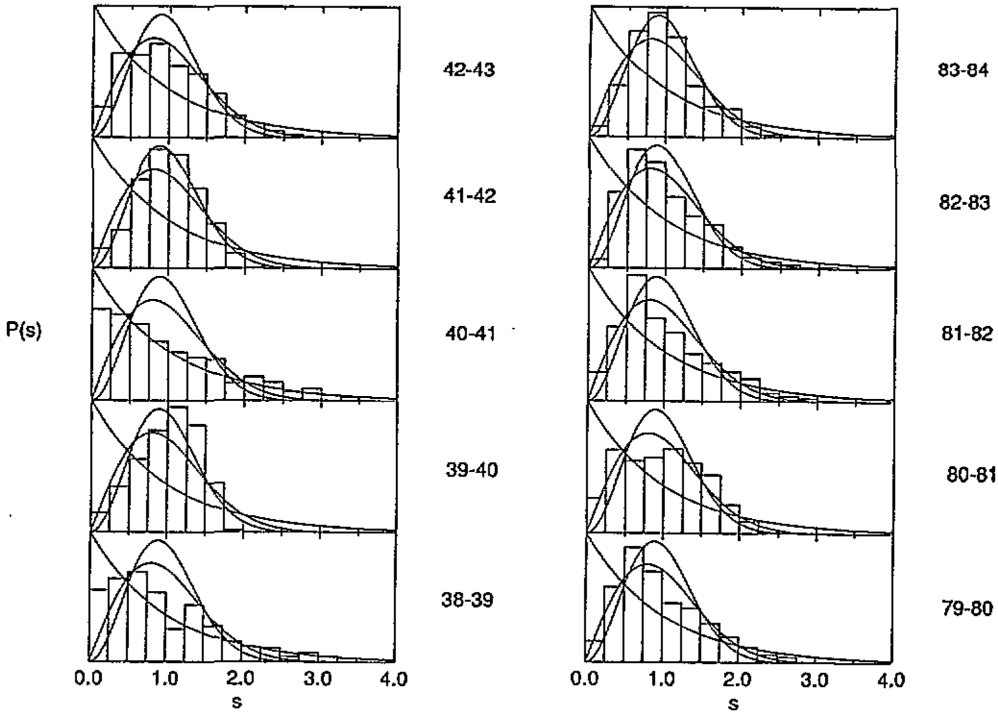

Figure 6. The level spacing distribution for different energy levels around the edge states (see figure 2(a)) for a $W = 2V$, $H = 0.2\phi_0/s^2$ sample. (a) Levels between the first and second Landau band. (b) Levels between the second and third Landau band.

Therefore, in this region one should not average the level spacing distributions over several level sequences, as is usually done in the Landau band region.

Acknowledgments

RB thanks the Alon Foundation and the US–Israel Binational Science Foundation for financial support.

References

[1] Gorkov L P and Eliashberg G M 1965 *Zh. Eksp. Teor. Fiz.* **48** 1407 (Engl. Transl. 1965 *Sov. Phys.–JETP* **21** 940)

[2] Altshuler B L and Shklovskii B I 1986 *Zh. Eksp. Teor. Fiz.* **91** 220 (Engl. Transl. 1986 *Sov. Phys.–JETP* **64** 127)

[3] Sivan U and Imry Y 1987 *Phys. Rev.* B **35** 6074

[4] Evangelou S N and Economou E N 1992 *Phys. Rev. Lett.* **68** 361

[5] Izrailev F M 1990 *Phys. Rep.* **129** 299

[6] Simmons B D and Altshuler B L 1993 *Phys. Rev.* B **48** 5422

[7] Shklovskii B I, Shapiro B, Sears B R, Lambrianides P and Shore H B 1993 *Phys. Rev.* B **47** 11 487

[8] Harris R and Yan Z 1993 *J. Phys.: Condens. Matter* **5** L493

[9] Hofstetter E and Schreiber M 1993 *Phys. Rev.* B **48** 16 979

[10] Kravtsov V E, Lerner I V, Altshuler B L and Aronov A G 1994 *Phys. Rev. Lett.* **72**

[11] Dupuis N and Montambaux G 1991 *Phys. Rev.* B **43** 14 390

[12] Huckestein B and Schewitzer L 1992 *High Magnetic Fields in Semiconductor Physics III* ed G Landwehr (Berlin: Springer)

[13] Ono Y, Kuwano H, Slevin K, Ohtsuki T and Kramer B 1993 *J. Phys. Soc. Japan* **62** 2762

[14] Ono Y and Ohtsuki T 1993 *J. Phys. Soc. Japan* **62** 3813

[15] Wigner E P 1951 *Ann. Math.* **53** 36; 1955 *Ann. Math.* **62** 548; 1957 *Ann. Math.* **65** 203; 1958 *Ann. Math.* **67** 325

[16] Dyson F J 1962 *J. Math. Phys.* **3** 140; 1962 *J. Math. Phys.* **3** 1157; 1962 *J. Math. Phys.* **3** 166

[17] Metha M L 1991 *Random Matrices* (San-Diego, CA: Academic)

[18] Pandry A and Metha M 1983 *Commun. Math. Phys.* **87** 449

[19] Van Wees B J *et al* 1989 *Phys. Rev. Lett.* **62** 1181

[20] Martin T and Feng S 1990 *Phys. Rev. Lett.* **64** 1971; 1991 *Phys. Rev.* B **44** 9084

[21] Hofstadter D R 1976 *Phys. Rev.* B **14** 2239

[22] Hatsugai Y 1993 *Phys. Rev.* B **48** 11 851

[23] Eliyahu D, Berkovits R, Abraham M and Avishai Y 1994 *Phys. Rev.* B at press

[24] Reichl L E 1992 *The Transition to Chaos in Conservative Classical Systems: Quantum Manifestations* (Berlin: Springer)

[25] Gutzwiller M C 1990 *Chaos in Classical and Quantum Mechanics* (Berlin: Springer)